Noise Figure: What is it and why does it matter?

A CRFS White Paper

Prepared by:

Jamie Hooper

CRFS Limited
Building 7200,
Cambridge Research
Park
Beach Drive,
Cambridge,
CB25 9TL, UK
Tel: +44 1223 859 500

CRFS Inc.
4230-D Lafayette Center
Drive
Chantilly
VA 20151
USA
Tel: +1 571 321 5470
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1 Introduction

Noise figure is one of the key parameters for quantifying receiver performance, telling you how low power a signal a receiver can detect. This White Paper will start from the basics to explain what exactly noise figure is, how to make sense of specifications and most importantly: why noise figure is important.

2 What is it?

Signal to noise ratio (SNR) is an essential indicator of if, and how easily, an RF signal can be distinguished from noise.

Any electrical system or subsystem, including radio receivers, will degrade SNR and this is quantified by a noise factor ratio of the SNR at the input and output. SNR measurements for calculating noise factor, are always taken at a standard temperature of 290 K.
Noise Figure: What is it and why does it matter?

\[ F = \text{Noise Factor} = \frac{S_i/N_i}{S_o/N_o} = \frac{SNR_{in}}{SNR_{out}} \]  

(1)

On electrical datasheets, you will usually see the term noise figure, this is simply noise factor converted to the logarithmic scale of decibels. A lower noise figure means less degradation of SNR and better performance, since smaller signals can be distinguished from noise.

\[ NF = \text{Noise Figure} = 10 \log_{10} F = 10 \log_{10} \frac{S_i/N_i}{S_o/S_i} \]  

(2)

3 Where does electrical noise come from?

Thermal Noise

Electrical noise is usually dominated by thermal noise, also known as Johnson-Nyquist noise. Thermal noise is an unavoidable consequence of the thermal motion of charge carriers in any electrical circuit. It is approximately white (power spectral density constant with frequency) and the level of thermal noise will hence increase proportionally with receiver resolution bandwidth setting. Thermal noise power is given by:

\[ P = K_B T B \]  

(3)

where \( K_B \) = Boltzmann’s constant, \( T \) = Temperature and \( B \) = Bandwidth.

This is more commonly expressed in logarithmic units of “dBm”, or decibels referenced to 1 mW. This is shown in the equation below, where \( 1 \times 10^{-3} \) is our 1 mW reference.

\[ P = 10 \log_{10} \left( \frac{K_B T B}{1 \times 10^{-3}} \right) \]  

(4)
Table 1 gives a few conversions from power in watts to power in dBm to give a sense of the scale for those unfamiliar with logarithmic units.

<table>
<thead>
<tr>
<th>Power (W)</th>
<th>Power (dBm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1 \times 10^{-21}$</td>
<td>$-180$</td>
</tr>
<tr>
<td>$1 \times 10^{-18}$</td>
<td>$-150$</td>
</tr>
<tr>
<td>$1 \times 10^{-15}$</td>
<td>$-120$</td>
</tr>
<tr>
<td>$1 \times 10^{-12}$</td>
<td>$-90$</td>
</tr>
<tr>
<td>$1 \times 10^{-9}$</td>
<td>$-60$</td>
</tr>
<tr>
<td>$1 \times 10^{-6}$</td>
<td>$-30$</td>
</tr>
<tr>
<td>$1$</td>
<td>30</td>
</tr>
</tbody>
</table>

**Table 1 Watt to dBm conversions**

For a 1 Hz bandwidth at a temperature of 290 Kelvin, equation -4 gives us a thermal noise power of $-174$ dBm or a PSD (Power Spectral Density) of $-174$ dBm/Hz. Variation over the range of sensible room temperatures is small enough that $-174$ dBm/Hz is assumed in all relevant spectrum analyser calculations.

Equivalent noise temperature is sometimes used to quantify noise power output from devices. In these cases, the temperature is not the actual temperature of the measured device, but the temperature that would create the same equivalent value of thermal noise as that measured at the device output.

**Other noise sources**

Other sources of electrical noise include shot noise related to discrete arrival times of electrons across barriers (such as those in diodes) and flicker noise associated with low frequencies (explanations for flicker noise are still the subject of much debate).

In spectrum analysers, non-thermal sources of electrical noise make a negligible contribution to the overall noise PSD.
4 What about phase noise?

Phase noise is a different category of receiver noise performance. It measures the small fluctuations in phase (or jitter) close to a signal carrier. While this is a completely separate category of noise to that addressed in this paper, the reader should bear in mind that for frequencies close to the carrier, phase noise will be the limiting factor on performance. The below charts show an example of jitter in the time domain for a simple carrier and how this translates to sidebands in the frequency domain.

<table>
<thead>
<tr>
<th>IDEAL – NO JITTER</th>
<th>JITTER</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time domain</td>
<td></td>
</tr>
<tr>
<td><img src="image1" alt="Image of IDEAL" /></td>
<td><img src="image2" alt="Image of JITTER" /></td>
</tr>
<tr>
<td>Frequency domain</td>
<td></td>
</tr>
<tr>
<td><img src="image3" alt="Chart of IDEAL" /></td>
<td><img src="image4" alt="Chart of JITTER" /></td>
</tr>
</tbody>
</table>

Figure 2: What phase noise looks like in the time and frequency domains
5 Making sense of datasheets

There are two dominant values used to communicate receiver noise performance: Noise Figure (NF) and Displayed Average Noise Level (DANL).

Noise figure, as explained above is a decibel measurement of how much the receiver degrades SNR. DANL is the noise power level displayed on a spectrum analyser as a result of such a noise figure.

The choice of parameter varies between manufacturers making comparison difficult. Fortunately, the below equation can be used for easy conversion between these parameters:

\[
DANL (dBm) = -174 + NF + 10 \log_{10}(BW)
\]  

where bandwidth BW corresponds to the Resolution Bandwidth (RBW) of the receiver.\(^1\)

Since DANL measurements are usually normalised to 1 Hz Resolution Bandwidth (RBW), this equation can be simplified to:

\[
DANL (dBm) = -174 + NF
\]  

Quoted best case values of noise figure for typical spectrum analysers are in the range of 6 to 30 dB with smaller values meaning better performance. Corresponding typical DANLs are around \(-168\) to \(-144\) dBm. Again, the smaller this number (the larger the magnitude of the negative value) the better the receiver’s noise performance.

Variation of gain with frequency for the various subsystems within the receiver will mean that both noise figure and DANL vary. A typical frequency characteristic of noise figure is shown below.

\(^1\) An additional scaling factor of approximately 1 to 2 of the resolution bandwidth may need to be included depending on the FFT windowing function used.
Figure 3: Noise figure frequency characteristic

As well as different NF or DANL values over different frequency ranges, noise performance is often stated for different preamplifier (also referred to as LNA – Low Noise Amplifier) ON/OFF settings and several attenuation settings.

Figure 4: Receiver preamplifier and attenuator

The best-case noise figure will be with 0 dB attenuation and the preamplifier switched on so that gain is at maximum. However, this maximum gain setup will not always be preferred. To avoid signal distortion or damage to circuitry from high power inputs and to maximise dynamic range (minimum to maximum signal power simultaneously measurable) gain will be reduced. A combination of switching off the preamplifier and increasing attenuation will be used for these high-power signals. The resulting rise in noise floor will not compromise signal detection as these high-power signals will still be well above the noise floor.
6 How do we measure it?

There are three key methods for measuring receiver noise figure as outlined in ITU-R SM.1838: Test procedure for measuring the noise figure of radio monitoring receivers. A summary of each method is provided here.

Gain Method

For a receiver at room temperature with a 50 Ω terminating input resistance such that the only input is $-174$ dBm/Hz noise we can assume output power density of:

$$P_{OUT} = -174 + NF + Gain$$  \hspace{1cm} (7)

If the gain of the receiver to be tested is known, the output power density can be measured and the equation rearranged for a noise figure (NF) value.

Since the output power will be very low for a good quality, low noise figure receiver, this method requires that the spectrum analyser used to measure the output power itself has a very low noise figure.

Figure 5: Gain method of noise figure measurement
Noise Figure: What is it and why does it matter?

Y-factor method

This method calculates receiver noise figure by using a noise source at the input switched on and off to generate two output noise powers. Noise figure can then be calculated from:

$$NF = 10 \log_{10} \left( \frac{10^{\text{ENR}/10}}{10^{Y/10} - 1} \right)$$  \hspace{1cm} (8)

where ENR is the ratio in dB of the noise source Power Spectral Density (PSD) ON/OFF and Y is the corresponding dB ratio of measured output PSD. A full derivation of equation 8 is provided in Appendix A.

Sensitivity Measurement

Noise figure can be determined indirectly from a measurement of receiver AM sensitivity (minimum signal required for given signal to noise and distortion ratio) using:

$$NF = S + 174 - 10 \log_{10}(\text{Res}) - 10 \log_{10} \frac{m^2}{1 + m^2}$$  \hspace{1cm} (9)

Where S=AM sensitivity, Res = Noise bandwidth and m= AM modulation index.

This is generally more complex than measuring noise figure directly, but may be useful if the AM sensitivity is already known.
7 How noise figure affects performance

Receiver noise figure or DANL directly translates into a limit on maximum detection distance and minimum detectable signal. If a receiver has a higher DANL, and hence a higher noise floor, then the received signal power will have to be higher in order to be distinguishable from noise as compared to a receiver with lower DANL, noise floor and noise figure. This means that for a transmitter at fixed distance, the transmitted signal power must be greater in order to be detectable and geolocatable. Equivalently, a transmitter at a given power would need to be closer to the receiver.

Processing gain may be used to allow detection of signals below the noise floor – for example by correlating two receiver measurements containing the signal to be detected. Assuming two receivers using the same signal processing methods are both able to detect x dB below their respective noise floors, comparison of DANL still provides a fair comparison between the two devices.

If for simplicity, we use DANL as the threshold for a signal being detectable, we can derive an equation for the relationship between detection range and DANL for a given transmitted signal power. Ignoring earth curvature and obstacle effects we can take the Friis transmission equation:

\[ P_R = \frac{P_T G_T G_R c^2}{(4\pi f)^2} \]  

where \( P_R \) = Power received in watts, \( P_T \) = Power transmitted in watts, \( G_T \) = Transmitter gain, \( G_R \) = Receiver gain, \( c \) = Speed of light, \( r \) = Distance and \( f \) = Frequency.

This equation shows how power received diminishes with the distance between transmitter and receiver and with increasing frequency. Both of these relationships are squared relationships i.e. if the distance between receiver and transmitter doubles, the received power reduces by a factor of 4.

We can substitute \( P_R \) for DANL in watts and rearrange to show how maximum detection distance for given transmitter power varies with minimum detectable power at the receiver (DANL).
Noise Figure: What is it and why does it matter?

\[ r = \sqrt{\frac{P_T G_T G_R c^2}{P_{\text{DANL}} (4\pi f)^2}} \]  

(11)

We then use this equation for two receivers with the same setup of receiver gain and transmitter gain, frequency and power. Taking a ratio between the two equations and simplifying, we derive an equation to show how improvement in DANL affects receiver detection range for a given signal:

\[ \frac{r_2}{r_1} = \sqrt{\frac{P_{\text{DANL1}}}{P_{\text{DANL2}}}} \]  

(12)

Where the “1” and “2” subscripts refer to two different receivers.

To get a real idea of what the dBm DANLs on datasheets mean for coverage range however, we need to convert our powers to dBm:

\[ \frac{r_2}{r_1} = \sqrt{\frac{10^{0.1 P_{\text{danl1}}}}{10^{0.1 P_{\text{danl2}}}}} = \sqrt{10^{0.1 (P_{\text{danl1}} - P_{\text{danl2}})}} \]  

(13)

With lower case “danl” used to differentiate powers measured in dBm from the previous watt measurements.

This means if receiver 2 has a DANL of −160 dBm compared to −150 dBm for the less sensitive receiver 1, then our detection range with receiver 2 will be approximately 3 (exactly $\sqrt{10}$) times greater. This means the coverage area with receiver 2 will be 10 times greater – in other words, you need fewer receivers for the same monitoring or geolocation task.
8 Why choose high performance

CRFS's RFeye spectrum monitoring systems offer the best performance against installation costs. This is achieved with high-performance, low noise figure hardware allowing fewer receivers to be used for coverage of a given area. Some manufacturers of cheap, low quality devices may claim their products offer better value for money, but the lower price tag is a false economy when the application requires the customer to buy 10 or even 20 times more units to achieve coverage of the same area.

A common argument in favour of using lots of low quality receivers rather than a few high-performance ones, is that obstacles will block a signal before its power is low enough for noise floor to be a limiting factor. The effect of obstacles on signal propagation means that in real-world scenarios, DANL improvement doesn’t translate into quite as large an improvement in coverage area as suggested by equation 13, but it is not generally a large deviation.

We can show this by using the simulation and modelling tools in our RFeye Software to visualise the effects of noise figure on receiver coverage in a real-world scenario. Table 2 shows a series of coverage heat maps for different values of noise figure. Each map is generated for an area of approximately 15 km by 20 km and detection of a 15 dBm 5 GHz signal.
### Table 2: Variation of coverage with noise figure

<table>
<thead>
<tr>
<th>Noise Figure (dB)</th>
<th>Coverage map</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td><img src="image1" alt="Coverage Map" /></td>
</tr>
<tr>
<td>12</td>
<td><img src="image2" alt="Coverage Map" /></td>
</tr>
<tr>
<td>18</td>
<td><img src="image3" alt="Coverage Map" /></td>
</tr>
</tbody>
</table>

The blue to red intensity scale of coverage represents the power level at the receiver relative to the noise floor (i.e., how easy a signal is to detect). The clear areas where the underlying satellite map is visible represent areas of no coverage. While we can see gaps in coverage due to obstacles, noise figure is still the limiting factor on coverage. The results show that a receiver with a 6 dB noise figure offers 12 times the coverage area compared to that with an 18 dB noise figure.

But don’t take our word for it, you can use equation 13 yourselves to quickly calculate what the “DANL” values on datasheets really mean for performance. (Use equation -6 to convert between noise figure and DANL where necessary). Appendix B provides a “Cutout and Keep Noise Figure Card”. This shows a to-scale diagram of relative coverage area for changes in noise figure and equation -6 for converting between DANL and noise figure.
If you would like more information on the importance of high-performance receivers and how our RFeye receiver technology allows smarter spectrum monitoring solutions, contact CRFS.
Appendices

A. Y-factor equation derivation

\[ ENR = N_{ON} - N_{OFF} \quad \text{and} \quad Y = N_{ON\,OUT} - N_{OFF\,OUT} \]

where \( N_{ON} \) and \( N_{OFF} \) are the input noise PSDs with the noise source on and off. \( N_{ON\,OUT} \) and \( N_{OFF\,OUT} \) are the corresponding output noise PSDs.

Converting these to linear equations in units of watts rather than decibels.

\[ ENR_W = \frac{N_{ON}}{N_{OFF}} \quad \text{and} \quad Y_W = \frac{N_{ON\,OUT}}{N_{OFF\,OUT}} \]

Since noise power in watts is proportional to noise temperature and the equivalent noise temperature with the noise source off is 290 K (associated with PSD of \(-174\) dBm/Hz:

\[ ENR_W = \frac{T_{ON} - 290}{290} \quad \text{and} \quad Y_W = \frac{T_{ON} + T_R}{290 + T_R} \]

where \( T_R \) is the noise temperature of the receiver being measured.

Rearranging the equation for \( Y \):

\[ Y_W = \frac{T_{ON} + T_R}{290 + T_R} \]

\[ = \frac{T_{ON} - 290 + 290 + T_R + 290 - 290}{290 + T_R} \]

Substituting \( ENR_W \) equation and equation for noise factor (\( F \)):

\[ F = \frac{T_R + 290}{290} \]

to give below:

\[ Y_W = \frac{ENR_W + 1 + F - 1}{F} = \frac{ENR_W}{F} + 1 \]
Rearranging for noise factor

\[ F = \frac{ENR_W}{Y_W - 1} \]

Converting \( ENR_W \) and \( Y_W \) to expressions in terms of original \( ENR \) and \( Y \) decibel measurements

\[ F = \frac{10^{ENR/10}}{10^{Y/10} - 1} \]

Converting to noise figure

\[ NF = 10 \log_{10} \frac{10^{ENR/10}}{10^{Y/10} - 1} \]

B. Cutout and Keep Noise Figure Card

Card-sized

Increasing coverage for 6 dB decrements in Noise Figure

\[ DANL (dBm) = -174 + NF \]

Over the page for full-sized version
Noise Figure: What is it and why does it matter?

Increasing coverage for 6 dB decrements in Noise Figure

\[ \text{DANL (dBm)} = -174 + \text{NF} \]
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Contact Information

CRFS Limited
Building 7200
Cambridge Research Park
Beach Drive
Cambridge
CB25 9TL
UK

Tel: +44 (0)1223 859500
Fax: +44 (0)1223 280351
Email: enquiries@crfs.com
Web: www.crfs.com